LA-UR-79-1074

CONF-790430--12

TITLE: MODELING AND SIMULATION FOR THE DESIGN AND
EVALUATION OF ADVANCED MATERIALS ACCOUNTING
SYSTEMS

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MASTER

SUBMITTED TO: European Safeguards Research and
Development Association Symposium,
April 25-27, 1979, Brussels, Belgium

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MODELING AND SIMULATION FOR THE DESIGN AND EVALUATION OF ADVANCED MATERIALS ACCOUNTING SYSTEMS*

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Abstract

Modeling and simulation techniques are used to design and evaluate advanced materials measurement and accounting systems for future nuclear fuel cycle facilities, using dynamic models of process and measurement operations. These techniques are required because operating data from modern facilities are not available. The Los Alamos safeguards modeling and simulation approach is discussed and is illustrated by a simple example.

I. Introduction

Effective safeguards control of special nuclear material (SNM) in a nuclear fuel cycle facility requires the ability to draw materials balances about the facility or portions of the facility. In the past, the accountability of nuclear materials and the detection of unauthorized removals have relied, almost exclusively, on discrete-item counting and materials-balance accounting following periodic shutdown, cleanout, and physical inventory. The classical materials balance associated with this system usually is drawn around the entire facility or a major portion of the process, and is formed by adding all measured receipts to the initial measured inventory and subtracting all measured removals and the final measured inventory. Although conventional materials-balance accounting is essential to safeguards control of nuclear material, it has inherent limitations in sensitivity and timeliness. Sensitivity is limited by measurement uncertainties that might obscure the diversion of a trigger quantity of SNM in a large throughput plant. Timeliness is limited by the infrequency of process shutdown, cleanout, and physical inventory; i.e., a loss of material could remain undiscovered until the next inventory is taken.

Safeguards effectiveness can be improved if conventional materials accounting procedures are augmented by dynamic materials accounting and control. The materials measurement and accounting system (MMAS) is designed for near-real-time control of SNM and incorporates the concepts of unit process accounting, dynamic materials balances, and graded safeguards. The goal of modeling and simulation of the process and measurement system for a nuclear facility is the design and subsequent evaluation of a MMAS that is based on a specific reference facility so that realistic quantitative conclusions can be reached.

II. Modeling and Simulation Approach

The design and evaluation of the MMAS must frequently be based on computer simulations of a

reference facility because neither the facility nor its safeguards system presently exist in readily modifiable form. Usually we are working with the design of nuclear facilities that are expected to be built some years in the future and rarely can we expect to change or to experiment with facilities already literally cast in concrete. Furthermore, the use of simulation techniques permits prediction of the dynamic behavior of materials flows under a wide range of operating parameters and accumulation of data quickly for relatively long operating periods. Alternative measurement strategies are readily compared and safeguards data analysis algorithms can be tested. In principle, the necessary data could be obtained from experiments on test loops and mock-ups of the plant operation, but this is both time consuming and expensive. Carefully selected test loops can be more effectively used to validate the computer models and to test portions of the final design of the MMAS.

Modeling and simulation of a facility and its MMAS constitute an essential part of the conceptual design of safeguards systems. MMAS design concepts are developed by (1) identifying key measurement points and appropriate measurement techniques; (2) comparing potential materials control strategies; (3) developing and testing appropriate data-analysis methods; and (4) quantitatively evaluating the MMAS's capability to detect diversion.

The modeling and simulation approach has been used extensively in safeguards concepts studies of chemical separations,[1] mixed-oxide fuel fabrication,[2] nitrate-to-oxide conversion[3] and coconversion,[4] and fast critical facilities.[5] This approach requires[6] (1) a detailed dynamic model of the process based on actual design data; (2) simulation of the model process on a digital computer; (3) a dynamic model for each measurement system; (4) simulation of accountability measurements applied to SNM flow and in-process inventory data generated using the model process; and (5) evaluation of simulated data from various materials accounting strategies.

III. Process Model

A detailed reference process design is required for making quantitative estimates of MMAS sensitivity because process variability, particularly in the levels of in-process inventory and material sidestreams (e.g., scrap, waste, and recycle), can have a significant effect on materials control. For example, the Barnwell Nuclear Fuels Processing Plant[7] at Barnwell, South Carolina (BNFP) was selected as the reference chemical separations facility; the model nitrate-to-oxide conversion facility was based on a reference design by Savannah River Laboratory and Savannah River Plant;[8,9] and the Westinghouse-Anderson design was selected as the reference facility for mixed-oxide fuel fabrication.[10]

Operation of a model process is simulated using standard Monte Carlo techniques developed for dynamic systems.[11,12] The dynamics of each process step are described by the continuity equation written for the flows of bulk material and SNM:

$$\dot{V} = \sum_i F_i ,$$

$$\dot{H} = \sum_i C_i F_i , \tag{1}$$

where

    i   = i^{th} material flow,
    F   = volumetric or mass flow rate of bulk
           material (inputs positive, outputs
           negative),
    C   = SNM concentration,
    V   = bulk volume or weight,
    H   = in-process inventory of SNM,
    (·) = time derivative, and
    Σ = summation over all materials flows.

The solution of the complete set of coupled differential equations for all process steps, subject to initial conditions and subsidiary constraints, describes the process dynamics. Random variation in the process is determined by the statistics selected for the independent process variables. The independent variables are selected after a detailed examination of the process and the operating procedures.

A computer code is developed to simulate the operation of the model process. The GASP IV simulation package[13] is used to schedule process events and to provide other routine services. GASP IV can execute both discrete-event and continuous time simulations.

Input data include initial values for all process variables and values of the statistical parameters that describe each independent, stochastic variable. Each process step is modeled separately. When an event is scheduled in a particular process step, the values of all concentrations, material transfers, and in-process inventory associated with that step are computed and stored in a data matrix. Simulated data from each process step are stored in separate data files. These data are available for further processing and as input to computer codes that simulate accountability measurements and materials balances.

IV. Measurement and Error Models

A. Measurements.

Operation of the materials measurement and accounting system is simulated using a Monte Carlo computer code. This code simulates measurements of the "true" materials flow data generated by the model process, calculates appropriate components of variance and covariance, and transmits appropriate measured values to the data-analysis code.[14]

The measurements simulation code incorporates both additive and multiplicative measurement-error models.[15]

1. **Additive Model.** In this model, the measured value m of a true quantity M is given by

$$m = M + \epsilon + \eta , \tag{2}$$

where ϵ is the error caused by instrument imprecision, and η is the error produced by uncertainty in the instrument calibration. Both errors are assumed to be independent and normally distributed with mean zero and variances σ_ϵ^2 and σ_η^2, respectively. The variance σ_m^2 of m is given by

$$\sigma_m^2 = \sigma_\epsilon^2 + \sigma_\eta^2 . \tag{3}$$

All measurements obtained from a given instrument using the same set of values for the calibration parameters are correlated through the calibration error η. The covariance σ_{ij} between the ith and j^{th} measured values is given by

$$\sigma_{ij} = \sigma_\eta^2 . \tag{4}$$

2. **Multiplicative Model.** In this model, the measured value m of a true quantity M is given by

$$m = M(1 + \epsilon + \eta) , \tag{5}$$

where ϵ is the relative error caused by instrument imprecision, and η is the relative error produced by uncertainty in the instrument calibration. Both errors are assumed to be independent and to be normally distributed with mean zero and variances σ_ϵ^2 and σ_η^2, respectively. The variance σ_m^2 of m is given by

$$\sigma_m^2 = M^2 \left(\sigma_\epsilon^2 + \sigma_\eta^2 \right) . \tag{6}$$

All measurement results obtained from a given instrument using the same set of values for the calibration parameters are correlated through the calibration error η. The covariance σ_{ij} between the ith and j^{th} measured values is given by

$$\sigma_{ij} = M_i M_j \sigma_\eta^2 . \tag{7}$$

3. **Measurement Simulation.** In the simulation, a value for η is periodically sampled from the appropriate distribution to coincide with the frequency of instrument recalibration. A value of ϵ is sampled for each measurement. Variance and covariance terms are estimated by replacing the true quantities M by the appropriate measured quantities. The precisions assigned to the measurements are based on estimates for similar instrumentation.

B. Materials Balances.

Each materials balance MB is a linear combination of measured quantities P_i of plutonium.

$$MB = \sum_{i=1}^{n} c_i P_i , \tag{8}$$

where c_i is +1 if P_i is an input or an initial in-process inventory and -1 if P_i is an output or final in-process inventory quantity for the accounting area. Often, the measured mass of plutonium is given by the product P of two different measured quantities.

$$P = x y , \qquad (9)$$

where x is either liquid volume or mass, and y is either plutonium concentration or mass fraction. The measurement-error model for both x and y is similar to that given in Eq. (2) or Eq. (5).

$$x = X + \epsilon_x + \eta_x \quad \text{and} \quad y = Y + \epsilon_y + \eta_y$$

for the additive model, or

$$x = X(1 + \epsilon_x + \eta_x) \quad \text{and} \quad y = Y(1 + \epsilon_y + \eta_y)$$

for the multiplicative model, where X and Y are true values, and the error components are defined as in Eq. (2) or Eq. (5). In some cases, the additive model may be appropriate for one of the measured quantities (X or Y), whereas the multiplicative model is appropriate for the other. In such cases, a combined model is used.

The variance σ_{MB}^2 in the computed value of a materials balance is a combination of the variances of the contributing measured values. Ordinarily, several measurements will be made using a given instrument before it is recalibrated, and appropriate correlation terms must be included in the computation of the materials balance variance. The general form of the equation used to estimate the variance of each materials balance is

$$\sigma_{MB}^2 = \sum_{i=1}^{n} x_i^2 \, Y_i^2 \left(\sigma_{\epsilon_x}^2 + \sigma_{\epsilon_y}^2 \right)$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \, X_i X_j \, Y_i Y_j \qquad (10)$$

$$\cdot \left(q_{ij} \, \sigma_{\eta_x}^2 + p_{ij} \, \sigma_{\eta_y}^2 \right) .$$

The quantity q_{ij} is +1 if X_i and X_j have been measured with a common value of η_x (that is, using the same instrument calibration) and is zero otherwise. Similarly, p_{ij} is +1 or 0, depending on whether a common value of η_y was used in the measurements of Y_i and Y_j. Each $i = j$ term in the double sum in Eq. (10) is the component of variance due to calibration uncertainty for a single measurement; these terms are present even if all measurements of X or Y are uncorrelated.

Equation (10) is written for the case where the multiplicative model is appropriate for the measurement of both X and Y. If the additive model applies to either measurement, the corresponding quantity (X or Y) is set to unity.

Measured values of net materials transfers and initial and final in-process inventories, along with appropriate components of variance and covariance, are computed by the measurement simulation code for each materials balance period. These quantities are sufficient for computation of materials balances, cusums, and the other test statistics described in Ref. 14.

In terms of the net transfer T (inputs positive, outputs negative), initial inventory I_i, and final inventory I_f, the materials balance equation becomes

$$MB = T + I_i - I_f . \qquad (12)$$

In some of the proposed accounting strategies, the terms in the computation of the materials balance variance, Eq. (10), arising from calibration errors in the inventory measurements approximately cancel because two inventory measurements appear with opposite signs in each materials balance equation. The magnitude of these terms is $(I_i - I_f)^2 \sigma_\eta^2$. In such cases, if the model process is operated near steady state so that $I_i \approx I_f$, the contribution to σ_{MB}^2 is relatively small; that is, calibration errors in the inventory measurements nearly cancel. In the error model, perfect cancellation is assumed for those in-process inventories in which $I_i \approx I_f$, and σ_η is set to zero for the associated in-process inventory measurements.

C. Cusums.

A cusum is computed after each materials-balance period. It is the sum of all materials balances for the unit process since the beginning of the accounting interval. The cusum variance is a complex combination of the variances of individual materials balances, because these balances usually are not independent. There are two principal sources of correlation between materials balances. The first is the correlation, discussed previously, between measurement results obtained using a common instrument calibration. The magnitudes of the associated covariance terms depend on the magnitude of the calibration error and the frequency of each instrument recalibration; omission of these terms can cause gross underestimation of the cusum variance. The second source of correlation between materials balances is the occurrence, with opposite signs, of each measured value of in-process inventory in two adjacent materials balances. As a result, only the first and last measurements of in-process inventory appear in the cusum, and only the corresponding variances appear in the cusum variance. The cusum variance is computed using an equation that has the same form as Eq. (10).

V. An Example

The modeling and simulation approach can best be illustrated by an example. For simplicity we will look at a single unit process.

A. The Model Process

In order to model and simulate the operation of a unit process, we must collect and examine its physical and operational characteristics.

The feed-blend tank of a coconversion process will be used as the sample model process.[16] The tank, illustrated in Fig. 1, is used to blend a coprocessed U/Pu nitrate solution from a chemical separations facility (stream 1) with a natural uranium nitrate solution (stream 2) to obtain the desired plutonium concentration (~10%). This tank will be used in parallel with
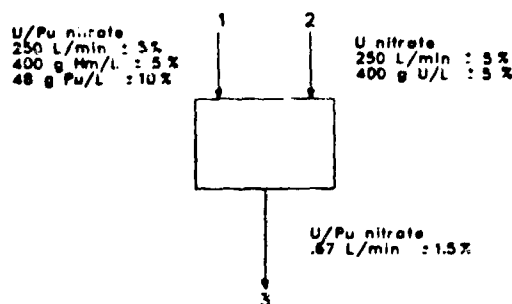
Fig. 1. The model process.

other similar tanks to feed a nitrate-to-oxide
coconversion process. The tank has a capacity
of 18 000 L and will usually be filled to about
16 000 L.

The operating procedures and assumptions
are:

1. The heavy metal and plutonium concentra-
   tions remain constant for each stream
   once the concentration is determined;
   i.e., it is assumed that solutions from
   the tanks feeding streams 1 and 2 are
   well mixed and that the feed-blend tank
   contents will be continuously mixed.
2. Once the feed-blend tank starts feeding
   the process it will continue until a
   heel of 1% ± 0.1% remains in the tank.
3. After the solution from chemical sepa-
   rations is received (14 000 L) the tank
   inventory will be sampled and analyzed
   to determine the U and Pu content so
   that the desired Pu concentration can
   be obtained by adding uranium nitrate.
4. After the uranium nitrate solution is
   added, the tank is sampled and a chemi-
   cal analysis is done. The tank will be
   isolated from the system until it is
   needed to feed the process.
5. Only one of the parallel streams will
   be operating at any given time.
6. The flow rate of solution into and out
   of the tank is linear over sufficiently
   small time steps.

The continuity equation (Eq. 1) must be
solved to model the dynamics of the process. The
amount of plutonium transferred in any given time
interval (t_1, t_2) is given by

$$S = C \int_{t_1}^{t_2} F(t) \, dt \ . \tag{12}$$

Assuming the flow is linear in the time interval
(assumption 6 above), the integral becomes

$$S = C \ \frac{F(t_1) + F(t_2)}{2} \ \Delta t \ , \tag{13}$$

where $\Delta t = t_2 - t_1$. A similar relationship
can be written for the heavy metal transfer (HM):

$$HM = C_{HM} \ \frac{F(t_1) + F(t_2)}{2} \ \Delta t, \tag{14}$$

where C_{HM} is the heavy metal concentration.

The solution of the continuity equation
(Eq. 1), over a time interval Δt, can now be
written. The change in the bulk mass (W) is
given by

$$W = (\overline{F1} \ \Delta t_1 + \overline{F2} \ \Delta t_2 - \overline{F3} \ \Delta t_3) + W(t_1)$$
$$- W(t_2) \ , \tag{15}$$

where $\overline{F1}$, $\overline{F2}$, and $\overline{F3}$ are the average mass flow
rates for streams 1, 2, and 3, respectively over
the period Δt, i.e.,

$$\overline{F} = \frac{F(t_1) + F(t_2)}{2} \ .$$

The change in the in-process inventory (H) of
plutonium is given by

$$H = C(W(t_1) - W(t_2))$$
$$+ C1 \ \overline{F1} \ \Delta t_1 - C3 \ \overline{F3} \ \Delta t_3 \ , \tag{16}$$

where C1 and C3 are the plutonium concentration
in stream 1 and 3, respectively (note that stream
2 has no plutonium), and W is the weight of the
tank. A similar equation can be written for the
change in the in-process inventory of heavy
metal by replacing the plutonium concentration
with the heavy metal concentration.

A computer code can now be written to sim-
ulate the operation of this process. When linked
with GASP IV and other service subroutines (in-
put, output, initialization, etc.), the combined
code produces the results described below.

B. Process Simulation.

Figures 2 through 4 are examples of SNM con-
centrations, transfers, and in-process inven-
tories from a day of simulated operation of this
unit process During this particular day, the
tank feeds the process until the desired heel is
reached; then it is refilled from the chemical
separations plant and the plutonium concentra-
tion is adjusted.

Figure 2 shows the plutonium concentration
and the solution mass in the tank for this day.
Note that the concentration is constant and the
weight slowly decreases until 11.5 h. At 11.5 h
the tank is almost empty and is switched out of
line (another tank would be switched in at the
same time); the feed from chemical separations
is started at that time, as shown in Fig. 3.
Note that the input flow rate is much larger than
the output flow rate; therefore, events occur
every 5 min instead of every 30 min. At 12.42 h
the dilution of the tank in-process inventory
starts, as shown in Fig. 4. By 12.58 h the tank
has been filled and will await assay and subse-
quent draining.

Figure 5 shows the plutonium concentration
and flow rate of the stream feeding the process
line during this example day. At 11.5 h the
concentration changes abruptly because feeding
tanks are switched.

C. The Measurement Model.

A measurement model must now be constructed
to convert the true values calculated by the
model process to observed values. The first step
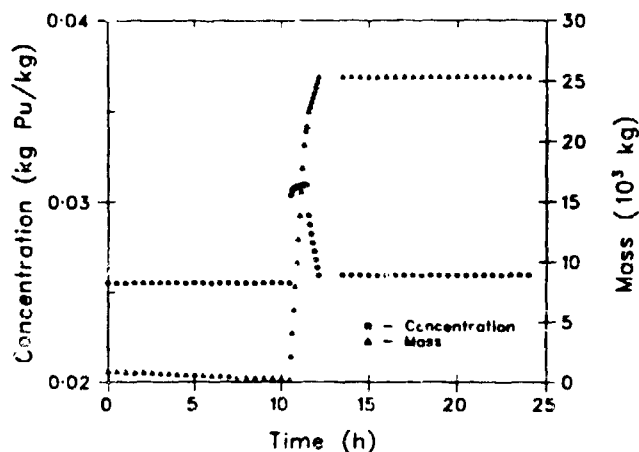is to write a materials-balance equation about
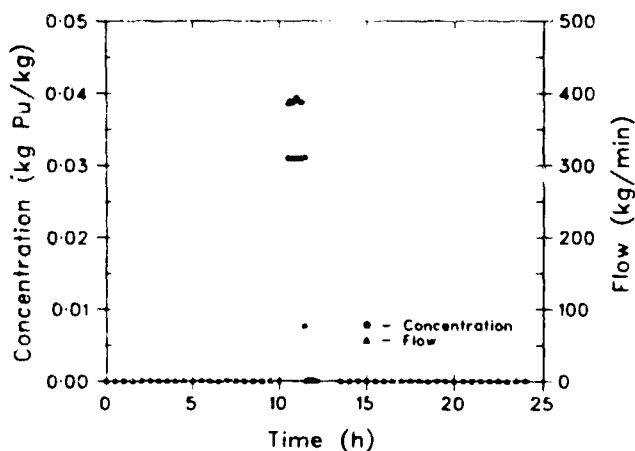
Fig. 2. Tank in-process inventory.

Fig. 4. Uranium nitrate feed.

Fig. 3. Feed from chemical separations.

Fig. 5. Process line feed.

each unit process accounting area in the facil-
ity. For this example a single unit-process
accounting area is formed about the tank.

1. Materials Balance. This unit process
has two inputs and a single output. These
streams do not operate simultaneously, so that
three separate materials-balance equations can
be written. Each materials balance will contain
an initial and final inventory and the net
amount of material that has been fed into or
taken out of the tank.

The materials balance for stream 1 (MB1) is

$$MB1 = C_o W_o + C1 \sum_{i=1}^{n} \overline{Fl}_i \, \Delta t_i - C_{f'} W_{f'} \quad (17)$$

where C_o = the in-process inventory initial
concentration, W_o = the in-process inventory
initial weight, and the subscript f' denotes
values at the time feed from chemical separations
is complete. This materials balance can be
rewritten

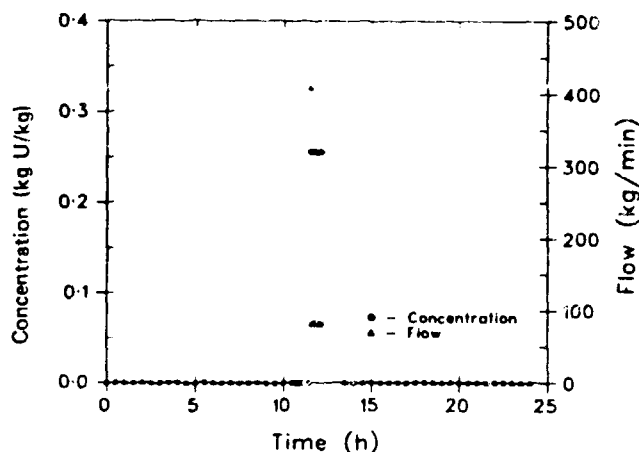$$MB1 = C_o W_o + T1 - C_{f'} W_{f'} \quad (18)$$

where

$$T1 = C1 \, W1,$$

and

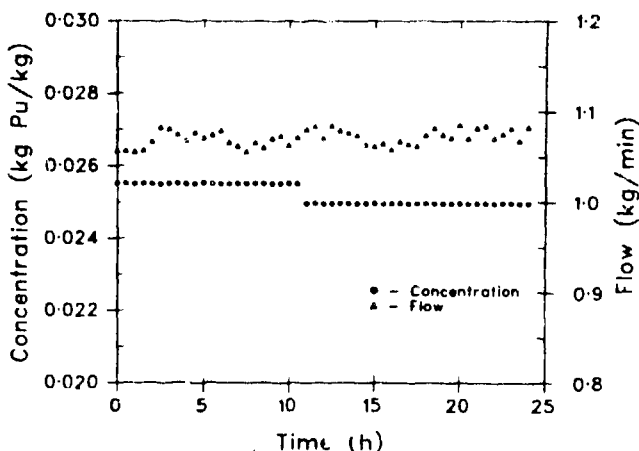$$W1 = \Delta t_1 \sum_{i=1}^{n} a_i \, Fl_i \, ,$$

where

$$a_0 = a_n = 1/2 \quad \text{and} \quad a_2 = \dots = a_{n-1} = 1 \, ,$$

i.e., T1 represents the total amount of SNM
transferred into the tank from chemical separa-
tions.

The materials balance for stream 2 (MB2) is

$$MB2 = C_{i'} W_{f'} - C_f W_f \, , \quad (19)$$

where the subscript f denotes values after dilu-
tion is complete. Note that stream 2 contains
no SNM.

The materials balance for stream 3 (MB3) is

$$MB3_i = C(\Delta W_i - T3_i) \, , \quad (20)$$

where C is the SNM concentration in the tank,

$$\Delta W_i = W_{i-1} - W_i \, ,$$

$$T3_i = \overline{F3}_i \, \Delta t_3 \, ,$$

and the subscript i denotes the time at which a materials balance is closed. In this example a materials balance is closed every 30 min.

2. **Materials Balance Variance.** The variance of the first materials balance is given by,

$$\sigma^2_{MB1} = C_o^2 \sigma^2_{w_o} + W_o^2 \sigma^2_{c_o} + C1^2 \sigma^2_{W1}$$

$$+ W1^2 \sigma^2_{c1} + C_{f'}^2 \sigma^2_{w_{f'}} + W_{f'}^2 \sigma^2_{c_{f'}}$$

$$+ 2W_o \ W1 \ COV(C_o, C_1)$$

$$- 2W_o \ W_{f'} \ COV(C_o, C_{f'}) \qquad (21)$$

$$- 2W1 \ W_{f'} \ COV(C_1, C_{f'})$$

$$- 2C_o \ C_{f'} \ COV(W_o, W_{f'}).$$

The three concentration measurements are correlated because all three chemical analyses will be done in the same laboratory using the same techniques and calibration standards. The weight measurements are correlated because frequent calibration of tank measurement apparatus is not expected. The additive error model is used for weight and the multiplicative error model is used for concentration; i.e., $w = W + \epsilon_w + n_w$ and $c = C(1 + \epsilon_c + n_c)$. Expanding the variance terms in the above equation leads to

$$\sigma^2_{MB1} = \left(c_o^2 + c_{f'}^2\right)\left(\sigma^2_{\epsilon_w} + \sigma^2_{n_w}\right)$$

$$+ \left(C_o^2 \ W_o^2 + C_{f'}^2 \ W_{f'}^2\right)\left(\sigma^2_{\epsilon_c} + \sigma^2_{n_c}\right)$$

$$+ C1^2 \ W1^2 \left(\sigma^2_{\epsilon_f} + \sigma^2_{\epsilon_c} + \sigma^2_{n_c}\right)$$

$$+ C1^2 \ \Delta t_1^2 \ \sigma^2_{n_f} \sum_o^n \sum_o^n a_i a_j \ F1_i F1_j$$

$$+ 2\sigma^2_{n_c} \left(W_o \ C_o \ T1 - W_o \ C_o \ W_{f'} \ C_{f'}\right. \qquad (22)$$

$$\left. - T1 \ C_{f'} \ W_{f'}\right) - 2C_o \ C_{f'} \ \sigma^2_{n_w} .$$

The variance equations of MB2 and MB3 are similar to σ^2_{MB1} and are given by

$$\sigma^2_{MB2} = \left(C_{f'}^2 + C_f^2\right)\left(\sigma^2_{\epsilon_w} + \sigma^2_{n_w}\right)$$

$$+ \left(C_{f'}^2 \ W_{f'}^2 + C_f^2 \ W_f^2\right)\left(\sigma^2_{\epsilon_c} + \sigma^2_{n_c}\right)$$

$$(23)$$

$$- 2C_{f'} \ C_f \ \sigma^2_{n_w} - 2C_{f'} \ C_f \ W_{f'} \ W_f \ \sigma^2_{n_c} ,$$

and

$$\sigma^2_{MB3_i} = C3_i^2 \left\{ \left(W_{i-1} - W_i - \overline{F3}_i \ \Delta t_3\right)^2 \right.$$

$$(24)$$

$$\cdot \left(\sigma^2_{\epsilon_c} + \sigma^2_{n_c}\right) + 2\sigma^2_{\epsilon_w}$$

$$\left. + \left(\frac{\Delta t_3}{2}\right)^2 \left[\sigma^2_{\epsilon_f} \left(F3_i^2 + F3_{i-1}^2\right) + \sigma^2_{n_f} \ \overline{F3}_i^2\right] \right\}.$$

3. **Cusum.** The cumulative summation for the sample day is a combination of all three materials balances and is a linear combination of the net transfers and the initial and final in-process inventories. After n materials balances,

$$CS_n = C_o \ W_o - C_f \ W_f + C1 \ \Delta t_1 \sum_o^n a_i \ F1_i$$

$$(25)$$

$$- C3 \ \Delta t_3 \sum_o^n a_i \ F3_i$$

where $C_o \ W_o$ is the initial plutonium in-process inventory, and $C_f \ W_f$ is the final plutonium in-process inventory.

4. **Cusum Variance.** The total variance of the cusum is a linear combination of the net transfer variances, the correlation between net transfers, the initial and final in-process inventory variances, and the correlation between the initial and final in-process inventories. The cusum variance is given by

$$\sigma^2_{CS_n} = \left(\sigma^2_{\epsilon_c} + \sigma^2_{n_c}\right)\left(C_o^2 \ W_o^2 + C_f^2 \ W_f^2 + C1^2 \ W1^2 + C3^2 \ W3^2\right) + \left(\sigma^2_{\epsilon_w} + \sigma^2_{n_w}\right)\left(C_o^2 + C_f^2\right) - 2 \ C_o \ C_f \ \sigma^2_{n_w}$$

$$(26)$$

$$+ C1^2\left(W1^2 \ \sigma^2_{\epsilon_f} + \Delta t_1^2 \ \sigma^2_{n_f} \sum_o^n \sum_o^n a_i \ a_j \ F1_i \ F1_j\right) + C3^2\left(W3^2 \ \sigma^2_{\epsilon_f} + \Delta t_3^2 \ \sigma^2_{n_f} \sum_o^n \sum_o^n a_i \ a_j \ F3_i \ F3_j\right)$$

$$+ 2\sigma^2_{n_c}\left(- C_o \ C_f \ W_o \ W_f + C_o \ C1_o \ W_o \ W1_o - C_o \ C3 \ W_o \ W3 - C_f \ C1 \ W_f \ W1 + C_f \ C3 \ W_f \ W3 - C1 \ C3 \ W1 \ W3\right).$$

Note that in the above equation it is assumed that $C_o \neq C_f \neq C1 \neq C3$. If the cusum is taken over a set of materials balances where some of the concentrations are equal then the variance of the cusum will change. For example, if the cusum is taken over materials balances for only stream 3 (the concentrations are all equal), then the cusum variance equation will be like Eq. (21). Extreme care must therefore be exercised in programming the measurement system in this case.

D. Measurement Simulation

The measurement errors that were used in this simulation are given in Table I. Note that the mass errors must be multiplied by the load-cell full-scale reading. Also note that the mass errors are in units of kilograms because an additive measurement model is used.

TABLE I

MEASUREMENT ERRORS

	(% 1σ)	(% 1σ)
Concentration	0.1	0.1
Mass	0.15	0.1

Figure 6 shows the materials balance chart and the cusum chart for the day when the tank feeds the process, is refilled, and the plutonium concentration is adjusted. Figure 7 shows the materials balance chart and cusum chart for a more typical day of operation in which the tank feeds the process all day. Note that the materials balances are almost never zero because of
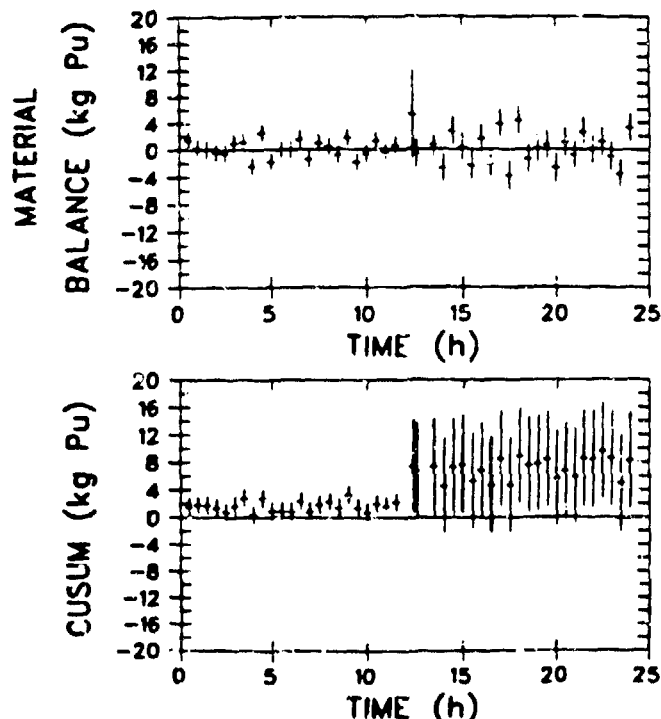
Fig. 6. Materials balance and cusum charts for one day-feed, fill, and adjust.
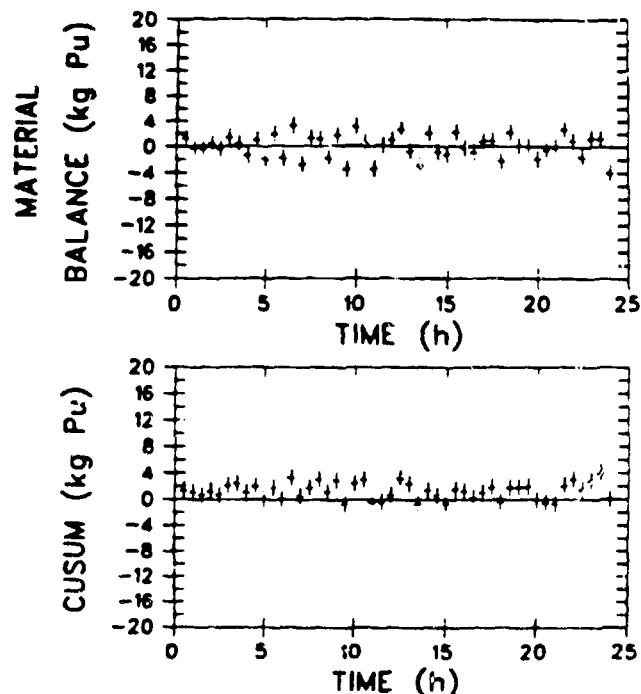
Fig. 7. Materials balance and cusum chart for one day-process line feed only.

measurement variability. The materials balance on Fig. 6, at 12.42 h, is drawn about the material transferred from chemical separations (see also Fig. 3). The large standard deviation results from the large amount of material transferred. From 12.42 h on, the error in this transfer dominates the cusum error. It is important to note that we do not have to take the cusum over that entire 24-h period. We could end the cusum at 12 h and start another cusum or examine the large transfer from chemical separations alone. This also suggests that it may be advantageous to close the materials balance by an inventory measurement before and after the transfer at the feed-blend tank and the chemical separations storage tank, thereby avoiding the relatively inaccurate flow measurements and the correlations between flow measurements.

The materials balance and cusum charts for a day when the tank is feeding the process (Fig. 7) show that the errors are significantly lower because we are not transferring as much material per balance period. Remember that Fig. 7 is more representative of normal operation because the tank feeds the process continuously over a two week period.

VI. Conclusions

Modeling and simulation techniques are valuable tools in the design and evaluation of materials measurement and accounting systems. These techniques are used to establish quantitative levels of sensitivity, to identify measurement control problems, and to test and evaluate different measurement strategies.

VII. Acknowledgment

VIII. References

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